International Journal of Applied Social Science
Volume 6 (1&2), January & February (2019) : 166-170
Received : 17.11.2018; Revised : 02.12.2018; Accepted : 18.12.2018

RESEARCH PAPER
ISSN : 2394-1405 (Print)

Construction of α-Resolvable 2-Associates PBIB Designs

BHARTI AGRAWAL*[1], RASHMI AWAD[2] AND SHAKTI BANERJEE[3]
[1,2,3]School of Statistics, Devi Ahilya University, Indore (M.P.) India

ABSTRACT

This paper deals with the construction method of α-resolvable group divisible (GD) designs with illustrations; which is based on the restricted dualisation of certain known affine resolvable balanced incomplete block designs.

Key Words : Balanced incomplete block design, Partially balanced incomplete block design, Group divisible design, α-resolvable design, Affine resolvable design, Affine α- resolvable, Dualisation

INTRODUCTION

A balanced incomplete block design is an arrangement of v symbols (treatment) into b sets (blocks) such that (i) each block contains k (< ν) distinct treatments; (ii) each treatment appears in r (> λ) different blocks and (iii) every pair of distinct treatments appears together in exactly λ blocks. Here, the parameters of balanced incomplete block design (ν, b, r, k, λ) are related by the parametric relations νr = bk, r (k-1) = λ (ν-1) and b ≥ ν (Fisher's inequality).

A block design is said to be resolvable if the b blocks each of size k can be grouped into r resolution sets of *b/r* blocks each such that in each resolution set every treatment is replicated exactly once. Bose (1942) proved that necessary condition for the resolvability of a balanced incomplete block design is b ≥ ν + r - 1.

A resolvable block design is said to be affine resolvable if and only if b=ν + r - 1 and any two blocks belonging to same resolution set have no treatment in common, say, q_1=0 whereas any two blocks belonging to different resolution sets intersect in the same number, say, $q_2=k^2/\nu$ of treatments.

The concept of resolvability and affine resolvability was generalized by Shirkhande and Raghavarao (1964) to α - resolvability and affine α- resolvability. An incomplete block design with parameters ν, b=βt, r = αt, k is said to be α-resolvable if the β blocks can be divided into t resolution sets of β blocks each, such that each treatment occurs α times in each resolution set. Further, α-resolvable incomplete block design is said to be affine α-resolvable if every two distinct blocks from the same resolution set intersect in the same number, say, q_1, of treatments, whereas every two blocks belonging to different α-resolution sets intersect in the same number, say, q_2, of treatments. The necessary and sufficient condition for the α - resolvable balanced incomplete block design to be affine α - resolvable with the block intersection numbers q_1 and q_2 is $q_1 = k(\alpha-1)/(\beta-1)$ and $q_2 = \alpha k/\beta = k^2/\nu$. There has been a very rapid development in this area of experimental designs. Some of the prominent work has been seen in Bailey *et al.* (1995), Banerjee *et al.* (1990), Kageyama (1972, 1973, 1977), Kageyama and Mohan (1983, 1984) Kageyama *et al.* (1989, 2001), Rai *et al.* (2004), Rudra *et al.* (2005), Agrawal *et al.* (2016).

A partially balanced incomplete block design based on an m-association scheme, with parameters *v, b, r, k,* λ_i (i = 1,2,…,*m*), is a block design with *v* treatments and *b* blocks of size *k* each such that every treatment occurs in *r* blocks and two distinct treatments being i^{th} associate occur together in exactly λ_1 (i=1, 2, ...m) blocks.

The existence of group divisible (GD) designs has been of interest over the years, going back to at least the work of Bose and Shimamoto (1952), who began classifying such designs. A group divisible (GD) design is a 2-associates partially balanced incomplete block design based on group divisible association scheme, *i.e.*

How to cite this Article: Agrawal, Bharti, Awad, Rashmi and Banerjee, Shakti (2019). Construction of α-Resolvable 2-Associates PBIB Designs. *Internat. J. Appl. Soc. Sci.,* **6** (1&2) : 166-170.

an arrangement of v = mn treatments in b blocks such that each block contains k (< v) treatments, each replicated r times, and the mn treatments can be divided into m groups of n treatments each, such that any two treatments occur together in λ_1 blocks if they belong to the same group and in λ_2 blocks if they belong to different groups. Furthermore, a group divisible (GD) design is said to be Singular (S) if $r - \lambda_1=0$; Semi regular (SR) if $r - \lambda_1>0$ and $rk - v\lambda_2 = 0$; Regular (R) if $r - \lambda_1 > 0$ and $rk - v\lambda_2 > 0$. For the definitions of partially balanced incomplete block design with their combinatorial properties, refer, Raghavarao (1971). Group divisible designs and partially balanced incomplete block designs have been studied by Banerjee and Kageyama (1986, 1988, 1990), Bhagwandas *et al.* (1985), Kageyama *et al.* (1989), Ghosh *et al.* (1989), Mohan and Kageyama (1983).

Bose and Nair (1939) first introduced the concept of "dualisation" in the field of design of experiments. They derived a new class of block designs by interchanging the role of treatments and blocks in a block design. Shrikhande (1952) applied the concept of dualisation to an asymmetric BIB design with λ = 1 or 2 to produce 2-associate PBIB designs. Dualisation of an incomplete block design with respect to unordered pairs of treatments could be found in Vanstone (1975), who constructed a BIB design through symmetric BIB designs with $\lambda \geq 2$. Using similar technique Mohan and Kageyama (1986) constructed 2-associate group divisible designs. The generalization of the concept due to Vanstone (1974) *i.e.* dualisation with respect to s- tuples could be seen in Kageyama and Mohan (1984) in constructing PBIB designs for any $s \geq 1$.

In the present paper a construction method of α-resolvable group divisible (GD) designs is provided, which is based on restricted dualisation of certain known affine α-resolvable balanced incomplete block designs. We have proposed 2-multiple solution of SRGD designs, which have no repeated blocks. Thus, theses designs, having three properties *i.e.* resolvability, multiplicity of design and no repeated blocks; which are interesting from statistical and/or combinatorial point of view and some new non-isomorphic solutions of the GD designs are also obtained. These non-isomorphic solutions (SR42, SR59 and SR76) are new in the sense that they are not reported in Clatworthy table (1973).

Method of Construction

Consider an affine α-resolvable balanced incomplete block design D, with the parameters v, $b=\beta t$, $r=\alpha t$, k, λ. Number the b blocks of design D by $B_1, B_2, \ldots\ldots B_b$ and dualise the design with respect to each of the treatment $\theta_1, \theta_2, \ldots\ldots\ldots\theta_v$. Thus, we get a semi regular group divisible (SRGD) design D_1 with the parameters $v'=b$, $b'=v$, $r'=k$, $k'=r$, $\lambda_1'=q_1$, $\lambda_2'=k^2/v$, $m'=t$ and $n'=b/t$. The association scheme of design D_1 is given as follows

$$\begin{array}{cccc} G_1 & G_2 & \cdots\cdots\cdots & G_{m'} \\ \hline \theta_1 & \theta_2 & \cdots\cdots\cdots & \theta_{n'} \\ \theta_{n'+1} & \theta_{n'+2} & \cdots\cdots\cdots & \theta_{2n'} \\ \vdots & \vdots & & \vdots \\ \theta_{2n'+1} & \theta_{2n'+2} & \cdots\cdots\cdots & \theta_{v'} \end{array} \qquad (1)$$

Now number the b blocks of the design D in reverse order by $B_b, B_{b-1}, \ldots\ldots\ldots, B_1$ and again dualise with respect to each of the treatment. By doing so, we get another SRGD design D_2 with the same parameters as of design D_1. The association scheme is also same as D_1. Then their juxtaposition given in the following structure.

$$D^* = [D_1 \vdots D_2] \qquad (2)$$

gives α-resolvable SRGD design D^* with the parameters $v^*=b$, $b^*=2v$, $r^*=2k$, $k^*=r$, $\lambda_1^*=2q_1$, $\lambda_2^*= 2(k^2/v)$, $m^*=t$ and $n^*=b/t$. The association scheme of the resultant design is given as follows :

$$\begin{array}{cccc} G_1 & G_2 & \cdots\cdots\cdots & G_{m^*} \\ \hline \theta_1 & \theta_2 & \cdots\cdots\cdots & \theta_{n^*} \\ \theta_{n^*+1} & \theta_{n^*+2} & \cdots\cdots\cdots & \theta_{2n^*} \\ \vdots & \vdots & & \vdots \\ \theta_{2n^*+1} & \theta_{2n^*+2} & \cdots\cdots\cdots & \theta_{v^*} \end{array} \qquad (3)$$

Theorem 2.1: The existence of affine α-resolvable balanced incomplete block design with parameters v, $b=\beta t$, $r=\alpha t$, k, λ implies the existence of α-resolvable SRGD design with the parameters $v^*=b$, $b^*=2v$, $r^*=2k$, $k^*=r$, $\lambda_1^*=2q_1$, $\lambda_2^*= 2(k^2/v)$, $m^*=t$ and $n^*=b/t$.

Proof: Consider an affine α-resolvable balanced incomplete block design D with parameters v, b, r, k, λ. Under the present construction method on dualising design D with respect to each of the treatments θ_1, θ_2, ... θ_v, we will get SRGD design D_1. The parameters $v'=b$, $b'=v$, $r' = k$ and $k'=r$, are obvious by construction.

Here there are $v'=b$ treatments, which are arranged in $m'=t$ groups of size $n'=b/t$ in such a way that any two treatments in the same group are first associates and any two treatments from different groups are second associates. The association scheme of the design D_1 is defined in (1).

Since in affine α-resolvable balanced incomplete block design; any two blocks from the same resolution set have $q_1 = k + \lambda - r$ treatments in common and any two blocks from different resolution set have $q_2 = k^2/v$ treatments in common. Thus in the design D_1 any (θ, ϕ) pair belonging to same group occurs together in $\lambda_1' = k + \lambda - r$ blocks ; whereas belonging to different groups occurs together in $\lambda_2' = k^2/v$.

Now number the b blocks of the design D in reverse order by B_b, B_{b-1}, ……..,B_1. Thus, we get another semi regular group divisible design D_2 with the same parameters as of design D_1; with no repeated blocks.

Further, juxtaposition the designs D_1 and D_2 as defined in (2), we will get the required design D^*. The parameters $v^*=b$, $b^*=2v$, $r^*=2k$, $k^*=r$, are obvious by construction method. In the resultant design, v^* treatments are arranged in m^* groups of size n^* each. After juxtaposition, any (θ, ϕ), pair belonging to same group occurs together in $\lambda_1^* = 2\,(k + \lambda - r)$ blocks; whereas belonging to different groups occurs together in $\lambda_2' = 2\,(k^2/v)$ blocks.

It is now obvious to note that in D^*; $r^*k^*-v^*\lambda_2^* = 0$. So, the resultant design is SRGD design. Also, there is a natural partition of resolution sets and every v^* treatment occurs r' times in each resolution set. Hence, the resultant design is r'-resolvable SRGD design. This completes the proof.

Corollary 2.2: When α = 1, then also Theorem 2.1 yields α-resolvable semi regular group divisible design with the parameters $v^*=b$, $b^*=2v$, $r^*=2k$, $k^*=r$, $\lambda_1^*=0$, $\lambda_2^* = 2\,(k^2/v)$, $m^*=r$ and $n^*=b/r$.

Example 2.3 : Consider an affine 1 - resolvable balanced incomplete block design D with parameters v=9, b=12, r=4, k=3 and λ=1. We number the b blocks of design D by 1,2,3,…..,12 as given below :

$$D = \left[\begin{array}{ccc|ccc|ccc|ccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \hline 1 & 4 & 7 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \\ 2 & 5 & 8 & 4 & 5 & 6 & 6 & 4 & 5 & 5 & 6 & 4 \\ 3 & 6 & 9 & 7 & 8 & 9 & 8 & 9 & 7 & 9 & 7 & 8 \end{array}\right]$$

After dualising design D, we will get the following SRGD design D_1 with parameters v'=12, b'=9, r'=3, k'=4, $\lambda_1'=0$, $\lambda_2'=1$, m'=4 and n'=3

$$D_1 = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 4 & 5 & 6 & 4 & 5 & 6 & 4 & 5 & 6 \\ 7 & 8 & 9 & 9 & 7 & 8 & 6 & 9 & 7 \\ 10 & 11 & 12 & 11 & 12 & 10 & 12 & 10 & 11 \end{bmatrix}$$

Now number the blocks of design D in reverse order by 12,11,……,1 as given below

$$D = \left[\begin{array}{ccc|ccc|ccc|ccc} 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ \hline 1 & 4 & 7 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \\ 2 & 5 & 8 & 4 & 5 & 6 & 6 & 4 & 5 & 5 & 6 & 4 \\ 3 & 6 & 9 & 7 & 8 & 9 & 8 & 9 & 7 & 9 & 7 & 8 \end{array}\right]$$

After dualising above design we will get following SRGD design D_2 with the same parameters as that of D_1 ; with no repeated block.

$$D_2 = \begin{bmatrix} 3 & 2 & 1 & 1 & 3 & 2 & 2 & 1 & 3 \\ 6 & 5 & 4 & 5 & 4 & 6 & 4 & 6 & 5 \\ 9 & 8 & 7 & 9 & 8 & 7 & 9 & 8 & 7 \\ 10 & 12 & 12 & 11 & 11 & 11 & 10 & 10 & 10 \end{bmatrix}$$

Then the construction method yields the following 3-resolvable SRGD design with parameters $v^*=12$, $b^*=18$, $r^*=6$, $k^*=4$, $\lambda_1^*=0$, $\lambda_2^*=2$, $m^*=4$ and $n^*=3$

$$D^* = \left[\begin{array}{ccccccccc|ccccccccc} 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 2 & 1 & 1 & 3 & 2 & 2 & 1 & 3 \\ 4 & 5 & 6 & 4 & 5 & 6 & 4 & 5 & 6 & 6 & 5 & 4 & 5 & 4 & 6 & 4 & 6 & 5 \\ 7 & 8 & 9 & 9 & 7 & 8 & 6 & 9 & 7 & 9 & 8 & 7 & 9 & 8 & 7 & 9 & 8 & 7 \\ 10 & 11 & 12 & 11 & 12 & 10 & 12 & 10 & 11 & 10 & 12 & 12 & 11 & 11 & 11 & 10 & 10 & 10 \end{array}\right]$$

The GD association scheme of the resultant design is given as follows

G_1	G_2	G_3	G_4
1	4	7	10
2	5	8	11
3	6	9	12

In the resultant design D^* every treatment is replicated 3 times in each resolution sets. Hence, the design constructed above is 3-resolvable SRGD design

Example 2.4: Consider an affine α-resolvable balanced incomplete block design D with parameters v=9, b=12, r=8, k=6, λ=5. We number the b blocks of design D by 1,2,3,…..,12 as given below

$$D = \left[\begin{array}{ccc|ccc|ccc|ccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \hline 4 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\ 5 & 2 & 2 & 3 & 3 & 2 & 3 & 3 & 2 & 3 & 3 & 2 \\ 6 & 3 & 3 & 5 & 4 & 4 & 4 & 4 & 5 & 4 & 5 & 4 \\ 7 & 7 & 4 & 6 & 6 & 5 & 6 & 5 & 6 & 5 & 6 & 6 \\ 8 & 8 & 5 & 8 & 7 & 7 & 7 & 8 & 7 & 7 & 7 & 8 \\ 9 & 9 & 6 & 9 & 9 & 8 & 8 & 9 & 9 & 9 & 8 & 9 \end{array}\right]$$

After dualising the design D, we will get the following SRGD design D_1 with parameters v'=12, b'=9, r'=6, k'=8, $\lambda_1'=3$, $\lambda_2'=4$, m'=4 and n'=3.

$$D_1 = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 2 \\ 5 & 4 & 4 & 5 & 4 & 4 & 5 & 4 & 4 \\ 6 & 6 & 5 & 6 & 6 & 5 & 6 & 6 & 5 \\ 8 & 7 & 7 & 7 & 8 & 7 & 7 & 7 & 8 \\ 9 & 9 & 8 & 8 & 9 & 9 & 8 & 8 & 9 \\ 11 & 10 & 10 & 10 & 10 & 11 & 11 & 11 & 10 \\ 12 & 12 & 11 & 12 & 11 & 12 & 12 & 12 & 12 \end{bmatrix}$$

Now number the blocks of design D in reverse order by 12,11,……,1 as given below

$$D = \left[\begin{array}{ccc|ccc|ccc|ccc} 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ \hline 4 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\ 5 & 2 & 2 & 3 & 3 & 2 & 3 & 3 & 2 & 3 & 3 & 2 \\ 6 & 3 & 3 & 5 & 4 & 4 & 4 & 4 & 5 & 4 & 5 & 4 \\ 7 & 7 & 4 & 6 & 6 & 5 & 6 & 5 & 6 & 5 & 6 & 6 \\ 8 & 8 & 5 & 8 & 7 & 7 & 7 & 8 & 7 & 7 & 7 & 8 \\ 9 & 9 & 6 & 9 & 9 & 8 & 8 & 9 & 9 & 9 & 8 & 9 \end{array}\right]$$

After dualising above design we will get following SRGD design D_2 with the same parameters as that of D_1 with no repeated block.

$$D_2 = \begin{bmatrix} 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 \\ 2 & 3 & 3 & 3 & 3 & 2 & 3 & 2 & 3 \\ 4 & 4 & 5 & 5 & 4 & 4 & 4 & 5 & 4 \\ 5 & 6 & 6 & 6 & 5 & 6 & 6 & 6 & 5 \\ 7 & 7 & 8 & 8 & 7 & 8 & 7 & 7 & 8 \\ 8 & 9 & 9 & 9 & 9 & 9 & 8 & 9 & 9 \\ 10 & 10 & 10 & 10 & 10 & 10 & 11 & 11 & 11 \\ 11 & 11 & 11 & 11 & 12 & 12 & 12 & 12 & 12 \end{bmatrix}$$

Then the construction method yields the following 6-resolvable SRGD design with parameters $v^*=12$, $b^*=18$, $r^*=12$, $k^*=8$, $\lambda_1^*=6$, $\lambda_2^*=8$, $m^*=4$ and $n^*=3$.

$$D^* = \left[\begin{array}{ccccccccc|ccccccccc} 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 \\ 3 & 3 & 3 & 3 & 3 & 3 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 2 & 3 & 2 & 3 \\ 5 & 4 & 4 & 5 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 5 & 5 & 4 & 4 & 4 & 5 & 4 \\ 6 & 6 & 5 & 6 & 6 & 5 & 6 & 6 & 5 & 5 & 6 & 6 & 6 & 5 & 6 & 6 & 6 & 5 \\ 8 & 7 & 7 & 7 & 8 & 7 & 7 & 7 & 8 & 7 & 7 & 8 & 8 & 7 & 8 & 7 & 7 & 8 \\ 9 & 9 & 8 & 8 & 9 & 9 & 9 & 8 & 9 & 8 & 9 & 9 & 9 & 9 & 9 & 8 & 9 & 9 \\ 11 & 10 & 10 & 10 & 10 & 11 & 10 & 11 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 11 & 11 & 11 \\ 12 & 12 & 11 & 12 & 11 & 12 & 11 & 12 & 12 & 11 & 11 & 11 & 11 & 12 & 12 & 12 & 12 & 12 \end{array}\right]$$

The GD association scheme of the above design is given as follows

G_1	G_2	G_3	G_4
1	4	7	10
2	5	8	11
3	6	9	12

In the resultant design D^* every treatment is replicated 6 times in each resolution sets. Hence, the design constructed above is 6-resolvable SRGD design.

RESULTS AND DISCUSSION

The following Table 1 provides the list of α-resolvable SRGD designs which are obtained by using certain known affine α-resolvable balanced incomplete block design.

Table 1 : List of α-resolvable SRGD designs

Sr. No.	Used Design						Resultant Design									Remark*
	Affine α-resolvable BIBD						α-resolvable SRGD design									
	v	b	r	k	λ	α	v^*	b^*	r^*	k^*	λ_1^*	λ_2^*	α	m^*	n^*	
1	9	12	8	6	5	2	12	18	12	8	6	8	6	4	3	
2	16	20	15	12	11	3	20	32	24	15	16	18	12	5	4	
3	25	30	24	20	19	4	30	50	40	24	30	32	20	6	5	
4	36	42	35	30	29	5	42	72	60	35	48	50	30	7	6	
5	49	56	48	42	41	6	56	98	84	48	70	72	42	8	7	
6(i)	9	12	4	3	1	1	12	18	6	4	0	2	3	4	3	SR42
7(ii)	16	20	5	4	1	1	20	32	8	5	0	2	4	5	4	SR59
8(iii)	25	30	6	5	1	1	30	50	10	6	0	2	5	6	5	SR76
9	36	42	7	6	1	1	42	72	12	7	0	2	6	7	6	
10	49	56	8	7	1	1	56	98	14	8	0	2	7	8	7	

(i) This design is being reported in Clatworthy (1973) as SR42. It is resolvable and duplicate plane of SR41. But here in our construction it is 3-resolvable SRGD design.

(ii) This design is being reported in Clatworthy (1973) as SR59. It is resolvable and duplicate plane of SR58. But here in our construction it is 4-resolvable SRGD design.

(iii) This design is being reported in Clatworthy (1973) as SR76. It is duplicate plane of SR75. But here in our construction it is 5-resolvable SRGD design.

*Other resultant designs have $r \geq 10$, so not reported in Clatworthy (1973).

REFERENCES

Agrawal, B. and Banerjee, S. (2016). Some constructions of 3-resolvable 2-associates PBIB designs. *ELIXIR Internat. J.*, **93** (C) : 39807-39813.

Bailey, R.A., Monod, H. and Morgan, J. P. (1995). Construction and optimality of affine-resolvable designs. *Biometrika*, **82** : 187–200.

Banerjee, S. and Kageyama, S. (1986). A method of constructing regular block designs. *J. Statst. Plann. Inf.,* **13** : 399-401.

Banerjee, S. and Kageyama, S. (1988). Some constructions of two and three - associate PBIB designs. *J. Japan Statst. Soc.,* **18** (2) :141-147.

Banerjee, S. and Kageyama, S. (1990). Existence of á-resolvable nested incomplete block designs. *Utilitas Math.,* **38** : 237–243.

Bhagwandas, Banerjee, S. and Kageyama, S. (1985). Patterned constructions of PBIB designs. *Commun. Statist. -Theor. Meth.,* **14** : 1259-1267.

Bose, R.C. (1942). A note on resolvability of balanced incomplete block designs. *Sankhya*, **6** : 105-110.

Bose, R.C. and Nair, K.R. (1939). Partially balanced block designs. *Sankhya*, **4** : 337-372.

Bose, R.C. and Shimamoto, T. (1952). Classification and analysis of partially balanced incomplete block designs with two associate classes. *J. American Statistical Association*, **47** : 151-184.

Clatworthy, W.H. (1973). Tables of two-associates-class partially balanced designs, NSB Applied Mathematics Series 63, U.S. Department of Commerce, National Bureau of standards, Washington, D.C.

Ghosh, D.K. Bhimani, G.C. and Kageyama, S. (1989). Resolvable semi-regular group divisible designs. *J. Japan Statst. Soc.,* **19** (2) : 163-165.

Kageyama, S. (1972). A survey of resolvable solutions of Balanced Incomplete Block Designs. *Internat. Statistical Review*, **40** (3) : 269-273.

Kageyama, S. (1973). On μ-resolvable and affine μ-resolvable balanced incomplete block designs. *Annals of Statistics*, **1** : 195–203.

Kageyama, S. (1977). Conditions for α-resolvability and affine α-resolvability of incomplete block designs, *J. Japan Statist. Soc.*, **7** : 19–25.

Kageyama, S. and Mohan, R. N. (1983). On μ- resolvable BIB designs, *Discrete Mathematics*, **45**, pp. 113-121, North – Holland Publishing Company.

Kageyama, S. and Mohan, R.N. (1984). Dualizing with respect to s-tuples. *Proc. Japan Acad. Ser. A,* **60** : 266-268.

Kageyama, S., Banerjee, S. and Verma, A. (1989). A construction of self complementary semi-regular group divisible designs. *Sankhya: Series B*, **51** : 335-338.

Kageyama, S., Majumder, A. and Pal, A. (2001). A new series of μ-resolvable BIB designs. *Bull. Grad. School Educ. Hiroshima Univ., Part II*, No. **50** : 41-45.

Mohan, R.N. and Kageyama, S. (1986). A method of construction of GD designs. *Utilitas Math.,* **24** : 311-318.

Mohan, R.N. and Kageyama, S. (1983). A method of construction of group divisible designs. *Utilitas Math.,* **24** : 311-316.

Philip, J., Banerjee, S. and Kageyama, S. (1997). Constructions of nested t-associate PBIB designs under restricted dualization. *Utilitas Mathematica*, **51** : 27-32.

Raghavarao, D. (1971). Constructions and Combinatorial Problems in Designs of Experiments. John Wiley, New York.

Rai, S., Banerjee, S. and Kageyama, S. (2004). A construction of resolvable nested 3-designs. *J. Combinatorial Designs,* **12** (6) : 466-470.

Rudra, S., Banerjee, S. and Kageyama, S. (2005). Construction of 3- resolvable nested 3-designs and 3-wise balanced designs. *Australasian J. Combinatorics*, **33** : 77-86.

Rudra, S., Banerjee, S. and Kageyama, S. (2006). A construction of resistant resolvable 3- designs without repeated blocks. *Utilitas Mathematica, South Africa*, **70** : 99-107.

Shrikhande, S.S. (1952). On the dual of some balanced incomplete block designs. *Biometrics*, **8** : 66-72.

Shrikhande, S.S. and Raghavarao, D. (1964). Affine á-resolvable incomplete block designs, Contributions to Statistics, Volume presented to Professor P. C. Mahalanobis on his 75th birthday, Pergamon Press, Oxford and Statistical Publishing Society, Calcutta, , 471–480.

Vanstone, A. (1975). A note on a construction of BIBD's. *Utilitas Math.,* **7** : 321-322.

****